Using Distributive Property to Enhance Multiplication Facts
(Partial Products)

Part One: The goal for Part One is that the students have an understanding of partial products and are able to draw a partial product area model independently. They can also begin to record the algebra portion.

There is a fully written script if that is needed and a **Summary of Partial Product Area Models for Part One**.

Part Two: In Part Two, the students continue to draw partial product area models using given expressions. By the end of the lesson, they record the algebra portion independently. If your students are doing well, you give them a second You Try, in which the students decide on their own how to decompose the 9.

Examples are shown in **Summary of Partial Product Area Models for Part Two**.

Next Steps: Once students understand how the partial product area model works, then they can try the generic rectangle and standard partial products models. Examples are provided on the **Partial Products: Next Steps** page.

Warm Up

Grade 3 Released Test Question:	**Current:**
What number makes this number sentence true? $3 + 5 = \square \times 2$ **A** 3 **B** 4 **C** 5 **D** 6 **Challenge:** Find what makes this equation true: $3 \times 5 = \square + 2$ How is this equation different from the one above?	Draw an area model to find the product of 4 x 5. **Challenge:** Draw a bar model to show the product of 4 x 5.

Review:	**Other:**
Write at least 5 different ways to decompose 9. **Challenge:** Write at least 2 ways to decompose 90.	Find the difference. Check with addition. 6,000 – 4,143 **Challenge:** Solve the above problem using a different method.

Warm Up (Debrief Page 1)

Grade 3 Released Test Question:

What number makes this number sentence true?

3 + 5 = [*4*] x 2

A 3

B 4

C 5

D 6

Challenge: Find what makes this equation true:

3 x 5 = [*13*] + 2

How is this equation different from the one above?

Debrief (Released Test Question)

Rel. Test Question:

3 + 5 (=) ___ x 2 ← Rewrite equation. Find the equals sign. Which side can be simplified?

8 (=) 4 x 2 ← Simplify 3 + 5. Think, What x 2 = 8? (4)

8 (=) 8 ← Simplify 4 x 2.

Now go back to the original equation and write 4 in the blank:

3 + 5 = *4* x 2.

Challenge:

3 x 5 (=) *(13)* + 2
15 (=) 13 + 2
15 (=) 15

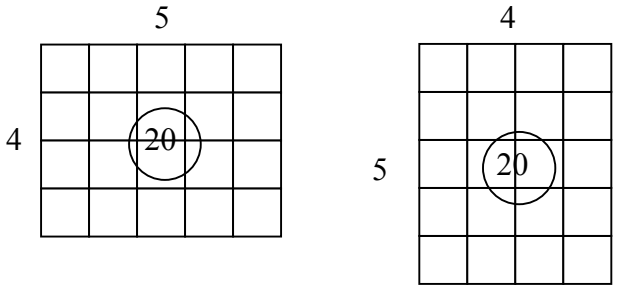
This equation is different from the first one because the operations have been switched. It's now 3 x 5 instead of 3 + 5 and ___ + 2 instead of ___ x 2.

Current:

Draw an area model to find the product of 4 x 5.

Challenge: Draw a bar model to show the product of 4 x 5.

Debrief (Current)



Challenge:

4 groups of 5 = 20

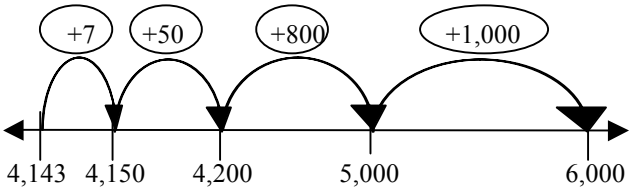
20			
5	5	5	5

(Multiplication is commutative, so this is also acceptable.)

20				
4	4	4	4	4

Warm Up (Debrief Page 2)

Review:	***Debrief (Review)***
Write at least 5 ways to decompose 9. **Challenge:** Write at least 2 ways to decompose 90.	*Examples Include:* 5 + 4 3 + 3 + 3 6 + 3 7 + 2 4 + 4 + 1 8 + 1 3 x 3 ***Challenge:*** Examples Include: 40 + 50 60 + 30 88 + 2 2 x 45

Other:	***Debrief (Other)***
Find the difference. Check with addition. 6,000 – 4,143 **Challenge:** Solve the above problem using a different method.	*Possible Method for Debrief:* 6,000 – 4,143 → 5,999 + 1 − 4,143 = 1,856 + 1 = 1,857 *Check:* 1 1 1 4,143 + 1,857 = 6,000 ***Challenge:*** *One Possible Alternate Method:* If 6,000 - 4,143 = ____, then 4,143 + ____ = 6,000  1,000 + 800 + 50 + 7 = 1,857

Lesson Plan for Part One

(Materials Needed: graph paper, pencils, crayons)

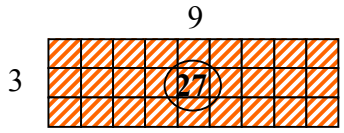
(After Warm Up has been debriefed…..)

Today we're going to learn a different way to think about x 9 facts. This method will help you get ready for algebra.

********** **Intro:** 3×9 **Basic Model** **********

Today we are going to work with: 3 x 9. Let's start by drawing an area model for 3 x 9.
(Students draw a 3 x 9 area model. They should be able to do this from previous experience.)

What is the product of 3 x 9? (27)



What if I don't know the product of 3 x 9? I can still figure it out by decomposing the more difficult factor, which is 9. I break the 9 down in to numbers that are easier to work with. *(Refer to Warm Up).*

**********Example 1: $3 \times (7 + 2)$—Teacher Model**********

One way is to decompose 9 is (7 + 2). Seven and two are easier for me to work with than 9. I'm going to write the first example, and you will do the second one with me.

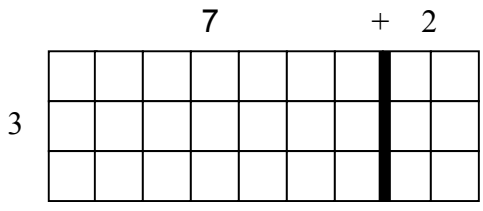
I'm going to write a new expression, substituting the (7+2) for 9. Thumbs up if you understand what I just did. These 2 expressions are equivalent. They have the same value. It's just that I have a 9 in the first one and a (7+2) in the second. Since 7+2 = 9, then the values are the same.

Now I am going to draw a new area model. I need to skip a couple spaces, so I have room to label the model. *(Draw the same area model, except label it as noted below.)*

(Picture and Notation so Far):

3 x 9

= 3 x (7 + 2)



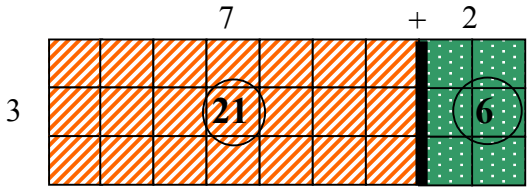
Ways to Decompose (9)
(7 + 2)
(8 + 1)
(5 + 4)
(6 + 3)
(4 + 4 + 1)

Now we will find the product of each section, or part.

(3 x 7 = 21—Color that section one color.)
(3 x 2 = 6 –Color that section a different color.)

The 21 and the 6 are called partial products. To get the whole product we need to combine them. What is 21 + 6? (27) Yes, it's 27. So…is 3 x 9 twenty seven? (yes)

(Completed Picture):



Now that we have finished our area model, and have figured out the whole product, we are going to *record,* or write down what we have done. That's the algebra part.

The first section we worked on was the 3 x 7 section, so I'm going to write (3 x 7) under that section. Under the next section, I'm going to write (3 x 2). We're going to combine these, so I'm going to write a + sign.

Now below that, I'm going to simplify. What is 3 x 7 again? (21) What is 3 x 2? (6)

Now below that, what is 21 + 6? (27) Great!

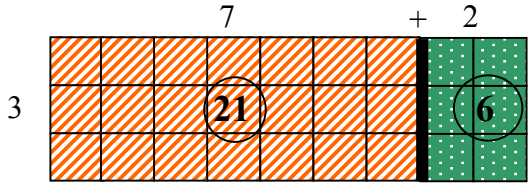
Now let's go back to the expressions: 3 x 9 and 3 x (7 + 2) *(read," 3 x the quantity 7 + 2")*

Did we get our whole product? Okay. I'm going to write my final answer at the bottom:
∴ 3 x 9 = 27 and 3 x (7 + 2) = 27

Final Picture with Algebra Notation for Example 1

3 x 9

= 3 x (7 + 2)



= (3 x 7) + (3 x 2)
= 21 + 6
= 27

∴ 3 x 9 = 27 and 3 x (7 + 2) = 27

********** **Example 2:** $3 \times (5 + 4)$ **Teacher/Student** **********

We will still work with 3 x 9, but we will decompose 9 in a different way. I want to see if we will still get a product of 27. Let's try (5+4). I like (5 + 4) because multiplying by 5 is easy for me. On your paper write the expression 3 x 9. Then beneath that, write 3 x (5 +4) because we are substituting the 9 for the quantity (5 + 4). Now think. Will we still get a product of 27? Raise your hand if you predict we will. Raise your hand if you predict we won't..

Now we draw our area model. *(Guide them through drawing and labeling factors.)*

Now think about the next step. Share your ideas with your partner. What is the next step? (Write the products.) Yes, we multiply 3 x 5 to get the product for one section, and 3 x 4 to get a product for the other section.*(Have them label and color each section for the products 15 & 12.)*

15 and 12 are our partial products. Say, "partial products."(partial products) How do we get our whole product? (add them) Yes we combine them. What is 15 + 12? (27)

Great! Now we record our work. To remember what the next step is, let's look back at the first example. Think about the first line we write under the picture. Share with a partner. Raise your hand if you can tell me. (We write (3 x 5) + (3 x 4)). Yes, we write (3 x 5) under the first section and (3 x 4) in the second.

Think about the next step. Look at the first example to give you a hint if you need to.
Share with your partner. Everyone tell me. (= 15 + 12) Yes.

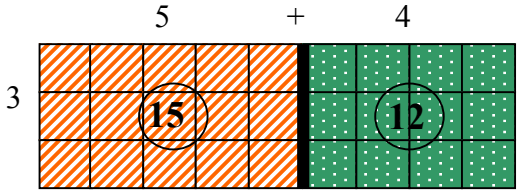
Think about the next step. Look at the first example to give you a hint if you need to.
Share with your partner. Everyone tell me. (= 27)

Great! Now look back at the original expression. 3 x (5 + 4)
What is our whole product? (27) Okay, now we can write, $\therefore$ 3 x (5 + 4) = 27

Raise your hand if you predicted we would get 27 as our product? (hands)
Great job! Raise your hand if you understand what we are doing.

Final Area Model and Algebra Notation for Example 2

3 x 9
= 3 x (5 + 4)



= (3 x 5) + (3 x 4)
= 15 + 12
= 27

$\therefore$ 3 x (5 + 4) = 27

********** **Example 3:** $3 \times (6 + 3) = 27$ **********

Student You Try for Picture/Some Assistance for Algebra

Raise your hand if you feel pretty confident about drawing these area models.

We will still work with 3 x 9. This time, we will decompose 9 in a different way to see if we will still get a product of 27. Let's look at our list to choose another expression. Share with your partner which one you would choose.

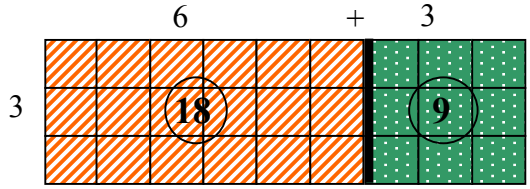
Let's try (6 + 3). I don't think 6 + 3 is as easy as 5+ 4, but I'd like to try it out. Raise your hand if you think (6+3) will be easier than (5+ 4). (hands) Harder. (hands)

What expression do we write? (3 x 9)
Below that, write 3 x (6 + 3).
What does the 6 + 3 stand for? (9)

You Try:

Now we need to draw our area model for 3 x (6 + 3). Skip a couple spaces first. Raise your hand if you think you can draw this area model all by yourself, including coloring the sections. Let's give that about 5 minutes.

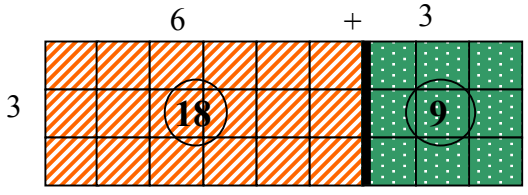
Debrief.



Great! Now we can record our work.
(Guide them through the algebra steps, one line at a time. Have them refer to their own notes if they need to. Then have them refer back to the original expression and write their final answers.)

Final Area Model and Algebra Notation for Example 3

3 x 9
= 3 x (6 + 3)



= (3 x 6) + (3 x 3)
= 18 + 9
= 27

$\therefore$ 3 x (6 + 3) = 27

**********Wrap Up**********

Remind me of the expression we started with today. (3 x 9)

Now take a look at the 3 ways we decomposed 9. Which one was easiest for you? Raise your hand for (7 + 2). (5 + 4) (6 + 3).

Raise your hand if you think you could do the picture on your own?

Raise your hand if you think you could to the algebra, or recording part, on your own?

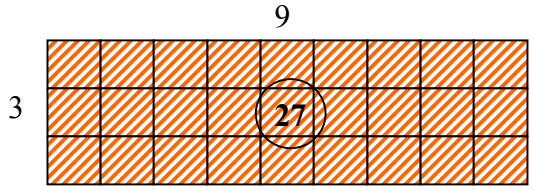
Tomorrow you will try the algebra, or recording part, on your own.

Thank you for all your hard work today.

Summary of Partial Product Area Models Part One

Basic Equation (Intro.)

3 x 9

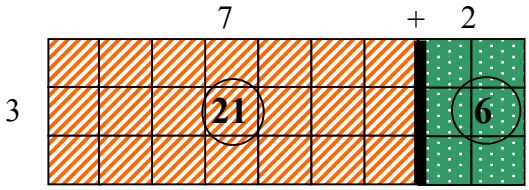


$\therefore$ 3 x 9 = 27

Example 1—Teacher Model

3 x 9

= 3 x (7 + 2)



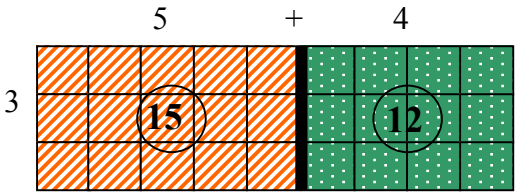
= (3 x 7) + (3 x 2)
= 21 + 6
= 27

$\therefore$ 3 x 9 = 27 and 3 x (7 + 2) = 27

Example 2—Teacher/Student

3 x 9

= 3 x (5 + 4)



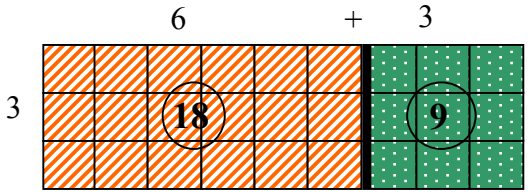
= (3 x 5) + (3 x 4)
= 15 + 12
= 27

$\therefore$ 3 x (5 + 4) = 27

Example 3—Student You Try for Picture. Some Assistance for Algebra

3 x 9

= 3 x (6 + 3)



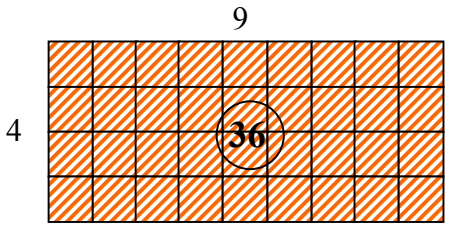
= (3 x 6) + (3 x 3)
= 18 + 9
= 27

$\therefore$ 3 x (6 + 3) = 27

Summary of Partial Product Area Models for Part Two

Basic Area Model (Intro)

4 x 9

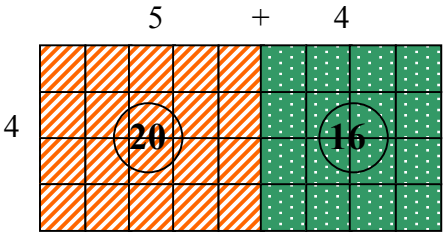


∴ 4 x 9 = 36

Example 1: Student You Try for Picture. Algebra with Some Teacher Assistance

4 x 9

= 4 x (5 + 4)



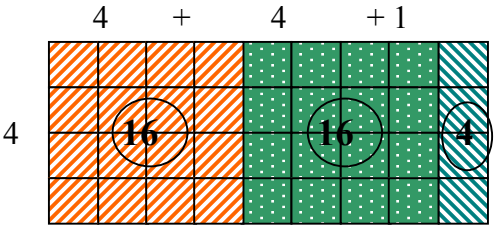
= (4 x 5) + (4 x 4)
= 20 + 16
= 36

∴ 4 x (5 + 4) = 36

Example 2: Introduce Three Addends

4 x 9

= 4 x (4 + 4 + 1)



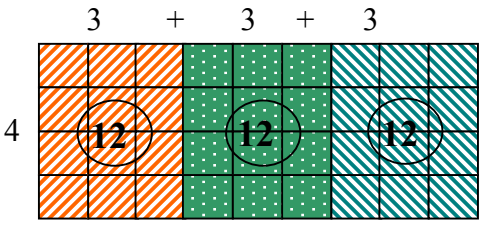
= (4 x 4) + (4 x 4) + (4 x 1)
= 16 + 16 + 4
= 36

∴ 4 x (4 + 4 + 1) = 36

Example 3: You Try

4 x 9

= 4 x (3 + 3 + 3)



= (4 x 3) + (4 x 3) + (4 x 3)
= 12 + 12 + 12
= 36

∴ 4 x (3 + 3+ 3) = 36

Partial Products—Next Steps

Once students understand how the partial products area model works, then they can try the generic rectangle and standard partial products models.

Generic Rectangle

7 x 9

	3	3	3
7	21	21	21

= 21 + 21 + 21

= 63

Partial Products

Method One:

$9 \rightarrow (3+3+3)$

$\times 7$ $\times 7$

21

21

+ 21

63

Method Two:

$9 \rightarrow (3+3+3)$

$\times 7$ $\times 7$

$21+21+21 = 63$

Generic Rectangle

8 x 8

	5	3
8	40	24

= 40 + 24

= 64

Partial Products

Method One:

$8 \rightarrow (5+3)$

$\times 8$ $\times 8$

40

+ 24

64

Method Two:

$8 \rightarrow (5+3)$

$\times 8$ $\times 8$

$40 + 24 = 64$